

VECTOR ANALYSIS PROBLEM SETS

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1. HAND IN: 28.09.21, COUNT: 10PTS.

1.1. Circle rolling on sine-graph. A circle with radius r rolls on top of the graph of the sine function. Find a parametric representation of the path traced out by the center of the circle. Assume that the radius r is sufficiently small, such that the circle travels through the valleys of the sine graph. Hint: Any graph $y = f(x)$ can be written as a parametric curve by using x as the parameter: $\vec{r}(x) = (x, f(x))$.

1.2. Charged particle in homogeneous magnetic field. The trajectory of a charged particle travelling through a magnetic field $\vec{B} = (0, 0, B)$ (with B a constant), is given by

$$\vec{r}(t) = \begin{pmatrix} R \cos(\omega t) \\ R \sin(\omega t) \\ ct \end{pmatrix},$$

where R , ω and c are constants.

- (i) Compute the length of the trajectory traced out between $t = 0$ and $t = 5T$, where $T = 2\pi/\omega$.
- (ii) Show that the given trajectory $\vec{r}(t)$ is in agreement with Newton's law of motion $\vec{F} = m\vec{a}$, where the acting force \vec{F} is given by $\vec{F} = q\vec{v} \times \vec{B}$, i.e. given by the Lorentz force.
- (iii) Find an expression for ω in terms of q , B , m .