Wiederholung: $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)\,dt \;;\quad b_n = \dots$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t} \;;\quad c_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega t}\,dt$$

Zeitverschiebung: $g(t) = f(t-a) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega(t-a)}$

$$= \sum_{n=-\infty}^{\infty} \tilde{c}_n e^{jn\omega t}$$

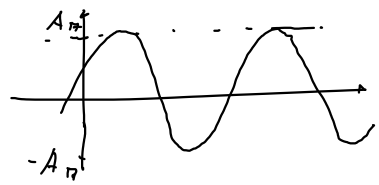
mit: $\underline{\tilde{c}_n = e^{-jn\omega a} c_n} \longrightarrow \underline{|\tilde{c}_n| = |c_n|}$

Wegen: $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos(n\omega t + \phi_n)$ (Amplituden / Phasen ...)

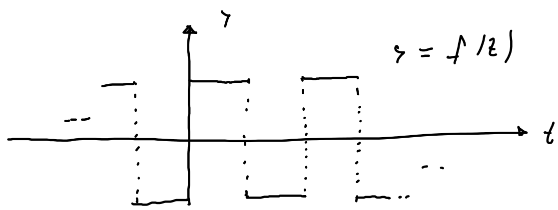
mit: $A_n = 2|c_n| \longrightarrow$ <u>Amplituden A_n ändern sich nicht!</u>

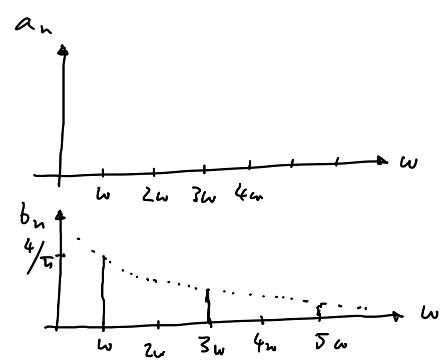
nur <u>Phasenverschiebung</u>!

$$\left[\; \underset{3}{\underset{\parallel}{a_{17}}} \cos(17\omega t) + \underset{2}{\underset{\parallel}{b_{17}}} \sin(17\omega t) = \underset{\sqrt{a_{17}^2 + b_{17}^2}}{\underset{\parallel}{A_{17}}} \cos(17\omega t + \phi_{17}) \;\right]$$



Spektrum: Bsp: Rechteck:





$\boxed{\text{Zeitbereich}}$ $\qquad$ $\boxed{\text{Frequenzbereich}}$

Frequenzänderung

Sei $f(t)$ eine T-per. Fkt.

Sei $g(t) = f\left(\frac{t}{a}\right) \longrightarrow g(t)$ ist aT-per. Fkt.

$f(t)$	$g(t)$
T	$\tilde{T} = aT$
ω	$\tilde{\omega} = \frac{\omega}{a}$

$$\longrightarrow \boxed{\tilde{a}_n} = \frac{2}{\tilde{T}} \int_0^{\tilde{T}} g(t) \cos(n \tilde{\omega} t)\, dt = \frac{2}{aT} \int_0^{aT} f\left(\frac{t}{a}\right) \cos\left(n\omega \frac{t}{a}\right) dt$$

$$\underset{u = \frac{t}{a}}{=} \frac{2}{\cancel{a}T} \int_0^{T} f(u) \cos(n\omega u)\, \cancel{a}\, du$$

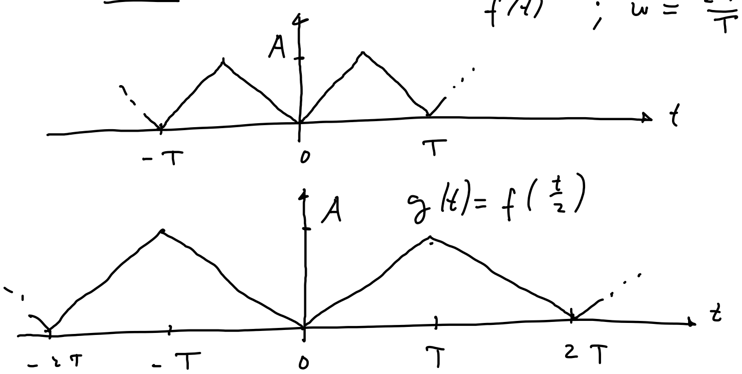
$$= \frac{2}{T} \int_0^{T} f(u) \cos(n\omega u)\, du = \boxed{a_n}$$

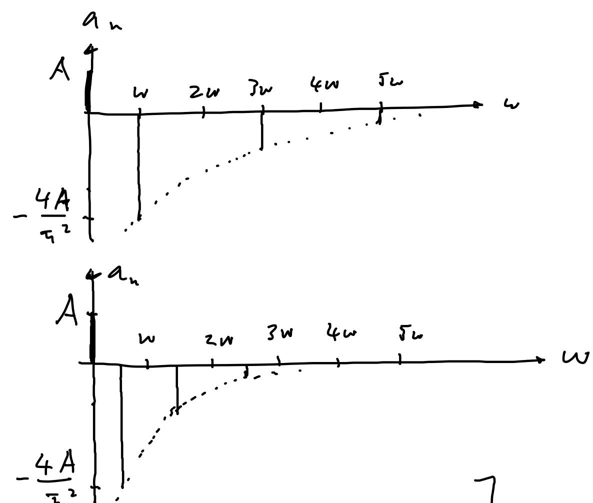
$\longrightarrow$ Fourierkoeff. ändern <u>nicht</u>!

Jedoch ändert sich die Frequenz: <u>Spektrum wird um Faktor $\frac{1}{a}$ gestreckt!</u>

<u>Bsp:</u> $a = 2$

$f(t)$; $\omega = \frac{2\pi}{T}$

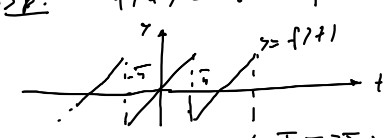




[⚠ ω ist Grundfrequenz von $f(t)$, <u>nicht</u> von $f\left(\frac{t}{2}\right)$!]

<u>Spektrum komplexe Form:</u>

<u>Bsp:</u> $f(t) = t$ für $t \in [-\pi, \pi]$



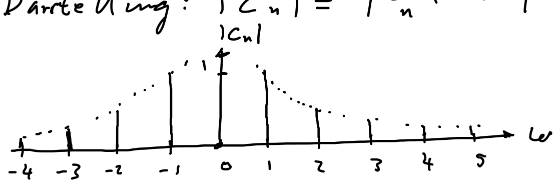
$\begin{pmatrix} T = 2\pi \\ \omega = 1 \end{pmatrix}$

$$\longrightarrow f(t) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{j}{n} (-1)^n e^{jnt}$$

I.e. $c_n = \frac{j}{n} (-1)^n : n \neq 0$

$c_0 = 0$

Grafische Darstellung: $|c_n| = \left| \frac{j}{n} (-1)^n \right| = \frac{1}{n}$

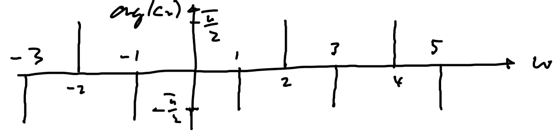


$|c_{-n}| = |\bar{c}_n| = |c_n|$

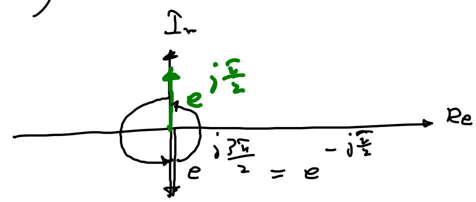
[$\arg(c_n)$: (Polarwinkel in $\mathbb{C}$-Ebene!) $\qquad -\pi \leq \arg(c_n) \leq \pi$

$$c_n = \frac{j}{n} (-1)^n = \frac{1}{n} e^{j\frac{\pi}{2}} \left(e^{j\pi}\right)^n$$

$$= \frac{1}{n} e^{j\left(\frac{\pi}{2} + n\pi\right)} \longrightarrow \arg(c_n) \overset{\downarrow}{=} \pm \frac{\pi}{2}$$



$$e^{j\left(\frac{\pi}{2}+n\pi\right)}$$



Aufgabe: $f(t) = \sin^3(3\pi t)$

Ges: FR in $\mathbb{C}$- Darstellung.

Lösg.: $\sin^3(3\pi t) = \left(\dfrac{e^{j3\pi t} - e^{-j3\pi t}}{2j}\right)^3 = -\dfrac{1}{8j}\left(e^{j9\pi t} - 3e^{j3\pi t} + 3e^{-j3\pi t} - e^{-j9\pi t}\right)$

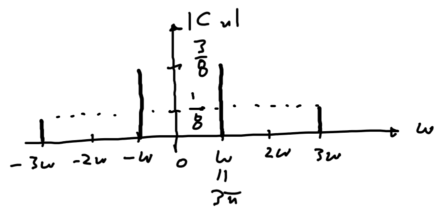
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\longrightarrow$ $\omega = 3\pi$

$c_3 = \frac{j}{8}$; $c_{-3} = -\frac{j}{8}$; $c_1 = -\frac{3j}{8}$; $c_{-1} = \frac{3j}{8}$

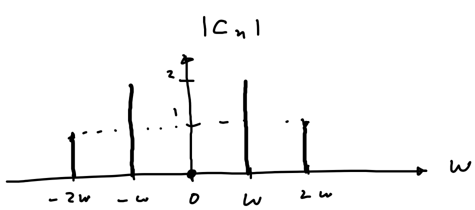
(alle anderen c_n's $= 0$!)

$\longrightarrow$ $|c_{\pm 3}| = \frac{1}{8}$; $|c_{\pm 1}| = \frac{3}{8}$



Aufgabe:

Sei komplexes Spektrum:



Ges: reelle, 2π-per. Fkt. $f(t)$ mit diesem Spektrum

Lösg.: Seien $\arg(c_n) = 0$; $T = 2\pi \rightarrow \omega = \frac{2\pi}{T} = 1$

$c_{-2} = 1$; $c_{-1} = 2$; $c_0 = 0$; $c_1 = 2$; $c_2 = 1$ (alle andern c_n's $= 0$)

$$\longrightarrow \underline{f(t)} = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t}$$

$$= e^{-j2\omega t} + 2e^{-j\omega t} + 2e^{j\omega t} + e^{j2\omega t}$$

$$\underset{\omega = 1}{=} e^{-j2t} + 2e^{-jt} + 2e^{jt} + e^{j2t}$$

$$= \underline{4\cos(t) + 2\cos(2t)}.$$