Fourierreihe

Eulerformeln

$$e^{jx} = \cos(x) + j\sin(x),$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}, \quad \cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$

$$T = \frac{2\pi}{\omega} \qquad \omega = 2\pi\nu$$

Fourierreihe von T-periodischen Funktionen

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \Big(a_n \cos(n\omega t) + b_n \sin(n\omega t)\Big)$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)dt \qquad a_0 = \frac{2}{T}\int_0^T f(t)dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)dt$$

Komplexe Darstellung

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t}, \qquad c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega t}dt$$

Umrechnung:

$$c_0 = \frac{a_0}{2}, \qquad c_n = \frac{1}{2}(a_n - jb_n),$$

$$c_{-n} = \frac{1}{2}(a_n + jb_n), \qquad n = 1, 2, 3, \ldots$$

Satz von Parseval

$$\frac{1}{T}\int_0^T f(t)^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

Fouriertransformation

$$\hat{f}(s) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi st}dt, \qquad \check{g}(t) = \int_{-\infty}^{\infty} g(s)e^{j2\pi st}ds$$

Sinc-Funktion

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Wahrscheinlichkeitsrechnung

Diskrete Zufallsvariablen

Binomialverteilung: $X \sim \mathcal{B}(n, p)$

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$E(X) = np$$

$$V(X) = np(1-p)$$

Stetige Zufallsvariablen

Normalverteilung: $X \sim \mathcal{N}(\mu, \sigma)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$E(X) = \mu$$

$$V(X) = \sigma^2$$

Standardnormalverteilung: $X \sim \mathcal{N}(\mu = 0, \sigma = 1)$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$E(X) = 0$$

$$V(X) = 1$$

Zentraler Grenzwertsatz

X_i, $i = 1, \ldots, n$ unabhängig, gleichverteilt mit

$$E(X_i) = \mu, \qquad V(X_i) = \sigma^2$$

Sei $S_n = \sum_{i=1}^n X_i$. Es gilt

$$\mu(S_n) = n\mu, \quad \sigma(S_n) = \sigma\sqrt{n}$$

und somit ist die Standardabweichung des Mittelwerts

$$\sigma\left(\frac{S_n}{n}\right) = \frac{\sigma}{\sqrt{n}}$$

Aussage zentraler Grenzwertsatz: Für

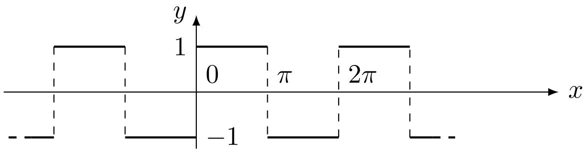
$$S_n^* = \frac{S_n - n\mu}{\sigma\sqrt{n}}$$

gilt

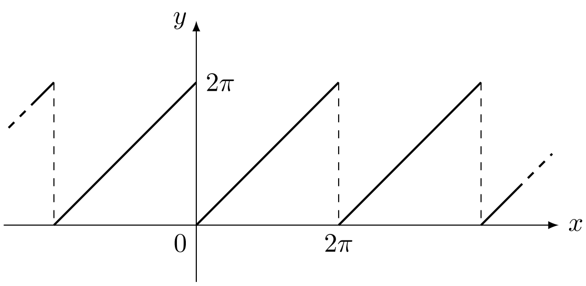
$$\lim_{n\to\infty} P(a \le S_n^* \le b) = \Phi(b) - \Phi(a)$$

Tabellen

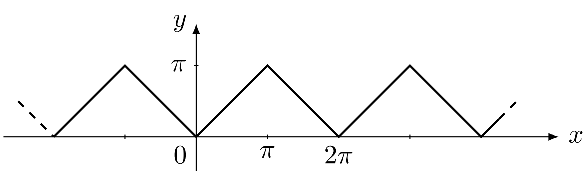
Wichtige Fourierreihen



$$f(x) = \frac{4}{\pi} \sum_{n \text{ ung.}} \frac{\sin(nx)}{n}$$

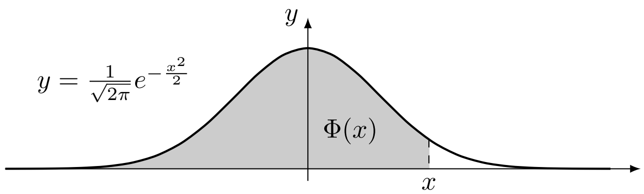


$$f(x) = \pi - 2 \sum_{n=1}^{\infty} \frac{\sin(nx)}{n}$$



$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n \text{ ung.}} \frac{\cos(nx)}{n^2}$$

Kumulative Verteilungsfunktion $\Phi(x)$ der Standardnormalverteilung



$\Phi(x)$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7793	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8906	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000