Eigenschaften der Laplacetransformation

$a > 0$

Ähnlichkeit.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "t\mapsto at"'] & F(s) \arrow[d] \\ f(at) \arrow[r, "\mathcal{L}"'] & \frac{1}{a}F\left(\frac{s}{a}\right) \end{tikzcd}$$

Dämpfung.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "\cdot e^{-at}"'] & F(s) \arrow[d, "s\mapsto s+a"] \\ e^{-at}f(t) \arrow[r, "\mathcal{L}"'] & F\left(s+a\right) \end{tikzcd}$$

Verschiebung.

$$\begin{tikzcd} H(t)f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "t\mapsto t-a"'] & F(s) \arrow[d, "\cdot e^{-as}"] \\ H(t-a)f(t-a) \arrow[r, "\mathcal{L}"'] & e^{-as}F\left(s\right) \end{tikzcd}$$

Ableitung Zeitbereich.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "\frac{d}{dt}"'] & F(s) \arrow[d] \\ f'(t) \arrow[r, "\mathcal{L}"'] & sF\left(s\right) - f(0^+) \end{tikzcd}$$

Ableitung Bildbereich.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "\cdot(-t)"'] & F(s) \arrow[d, "\frac{d}{ds}"] \\ -tf(t) \arrow[r, "\mathcal{L}"'] & F'(s) \end{tikzcd}$$

Integral Zeitbereich.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d] & F(s) \arrow[d, "\cdot\frac{1}{s}"] \\ \int_0^t f(u)du \arrow[r, "\mathcal{L}"'] & \frac{1}{s}F(s) \end{tikzcd}$$

Integral Bildbereich.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "\cdot\frac{1}{t}"'] & F(s) \arrow[d] \\ \frac{1}{t}f(t) \arrow[r, "\mathcal{L}"'] & \int_s^{+\infty} F(u)du \end{tikzcd}$$

Faltung.

$$\begin{tikzcd} H(t)f_1(t) \arrow[r] \arrow[d, "\mathcal{L}"'] & (f_1 * f_2)(t) \arrow[d, "\mathcal{L}"'] & H(t)f_2(t) \arrow[l] \arrow[d, "\mathcal{L}"'] \\ F_1(s) \arrow[r] & F_1(s)\cdot F_2(s) & F_2(s) \arrow[l] \end{tikzcd}$$

$$(f_1 * f_2)(t) = \int_0^t f_1(u)f_2(t-u)du, \quad \text{für} \quad t \geq 0.$$

Transformationen

$f(t)$	$F(s) = \mathcal{L}(f(t))$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin(t)$	$\frac{1}{s^2+1}$
$\cos(t)$	$\frac{s}{s^2+1}$
$\delta(t)$	1
$f(t+T) = f(t)$	$\frac{1}{1-e^{-sT}} \int_0^T f(t)e^{-st}dt$