Wiederholung: Frequenzantwort

Stabiles LTI-System $G(s) = \frac{Z(s)}{N(s)}$ mit Eingang: $u(t) = e^{j\omega t}$

$$\longrightarrow U(s) = \frac{1}{s - j\omega} \longrightarrow Y(s) = G(s)\,U(s)$$

$$= \frac{Z(s)}{N(s)(s-j\omega)} = \frac{a}{s - j\omega} + \text{Rest} \qquad (*)$$

$$\mathcal{L}^{-1}(\text{Rest}) \xrightarrow{t \to \infty} 0$$

$$(*)\cdot(s-j\omega) \longrightarrow \frac{Z(s)}{N(s)} = a + (s-j\omega)\cdot\text{Rest}$$

$$s = j\omega \longrightarrow \boxed{a = \frac{Z(j\omega)}{N(j\omega)} = G(j\omega) = |G(j\omega)|\, e^{j\varphi(\omega)}}$$

$$\varphi = \arg(G(j\omega))$$

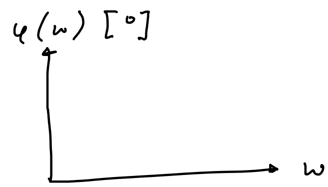
$$y(t) = \mathcal{L}^{-1}(Y(s)) \underset{(\text{"}t\text{ gross"})}{=} a\, e^{j\omega t} = G(j\omega)\, e^{j\omega t}$$

$$= |G(j\omega)|\, e^{j(\omega t + \varphi(\omega))}$$

$$\left.\begin{array}{ll} |G(j\omega)|: & \text{Verstärkungsfaktor} \\ \varphi(\omega): & \text{Phasenverschiebung} \end{array}\right\} \text{Frequenzgang}$$

$\longrightarrow$ Frequenzgang gegeben durch Transferfkt. $G(s)$ entlang imag. Achse ||

Grafische Darstellung: Bode Plots

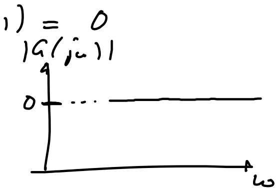


dB: $20 \log_{10}(|G(j\omega)|)$

Bsp.: $G(s) = \frac{s}{s+1}$: $|G(j\omega)| = \left|\frac{j\omega}{j\omega + 1}\right|$

ω gross: $|G(j\omega)| = \left|\frac{j\omega}{j\omega+1}\right| \approx 1 \longrightarrow \log_{10}(|G(j\omega)|) \approx \log_{10}(1) = 0$

$\longrightarrow 20 \log_{10}(|G(j\omega)|) \approx 0$

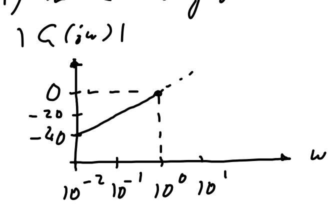


ω klein: $|G(j\omega)| = \left|\frac{j\omega}{j\omega+1}\right| \approx \omega \longrightarrow \log_{10}(|G(j\omega)|) \approx \log_{10}(\omega)$

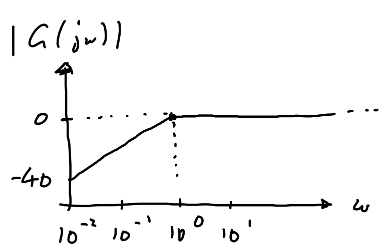
$j\omega + 1 \approx 1$

$\longrightarrow 20 \log_{10}(|G(j\omega)|) \approx 20 \log_{10}(\omega)$

Gerade mit Steigung 20



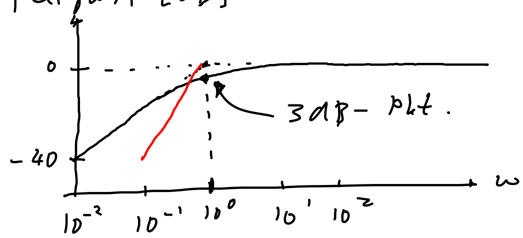
Zusammen:



Im Schnittpkt: $\omega = 10^0 = 1$

$$\longrightarrow |G(j\omega)| = \left|\frac{j\omega}{j\omega+1}\right| = \left|\frac{j}{j+1}\right| = \frac{|j|}{|j+1|} = \frac{1}{\sqrt{2}}$$

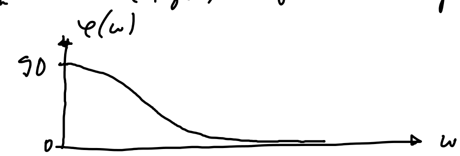
$$\longrightarrow 20 \log_{10}\left(|G(j\omega)|\right) = 20 \log_{10}\left(2^{-\frac{1}{2}}\right) = -10 \underbrace{\log_{10}(2)}_{0.301} = \underline{-3.01}$$



Phase:

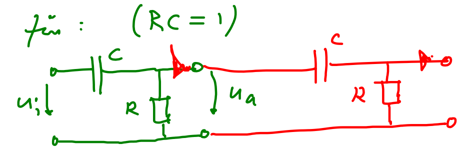
ω gross: $G(j\omega) \approx 1 \longrightarrow \arg(G(j\omega)) \approx 0$

ω klein: $G(j\omega) \approx j\omega \longrightarrow \arg(G(j\omega)) \approx \frac{\pi}{2} = 90°$



Bem: $G(s) = \frac{s}{1+s} \cdot \frac{s}{1+s}$

ist Transferfkt. für: $(RC = 1)$



Hochpass!

Aufgabe: $G(s) = \frac{1}{s+1}$

Ges.: Bode-Plot für $|G(j\omega)|$

Lösg.: ω gross: $|G(j\omega)| = \left|\frac{1}{j\omega+1}\right| \approx \frac{1}{\omega}$

$$20\log_{10}(|G(j\omega)|) \approx 20 \log_{10}\left(\frac{1}{\omega}\right)$$
$$= -20 \log_{10}(\omega)$$

$\longrightarrow$ Gerade mit Steigung $= -20$

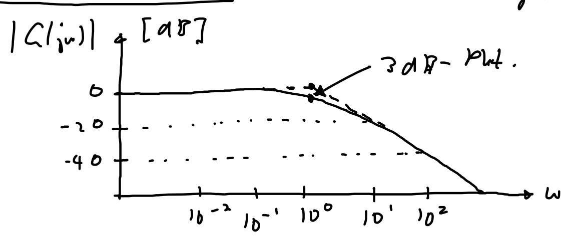
ω klein: $|G(j\omega)| = \left|\frac{1}{j\omega+1}\right| \approx 1$

$$20\log_{10}(|G(j\omega)|) \approx 0$$

Schnittpkt. der Asymptoten bei: $\omega = 10^0 = 1$
$(\rightarrow \log_{10}(\omega) = 0)$

3dB-Pkt. bei $\omega = 10^0 = 1$: $\longrightarrow |G(j\omega)| = \left|\frac{1}{j+1}\right| = \frac{1}{\sqrt{2}} \rightarrow 20 \log_{10}(|G(j\omega)|) = 20 \log_{10}\left(\frac{1}{\sqrt{2}}\right)$

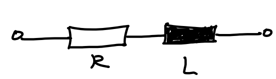
$$= -10 \log_{10}(2)$$
$$= \underline{-3.01}$$



(Tiefpass)

Ortskurven: Ist Darstellung einer komplexen Grösse, welche von einem reellen Parameter abhängt.

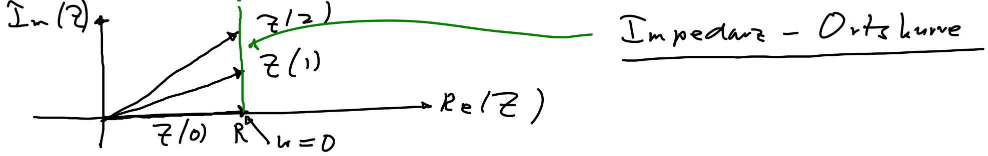
Bsp: Impedanz Z von:



$$Z = Z(\omega) = R + j\omega L \qquad \omega \geq 0$$

ω reeller Parameter: Kreisfrequenz

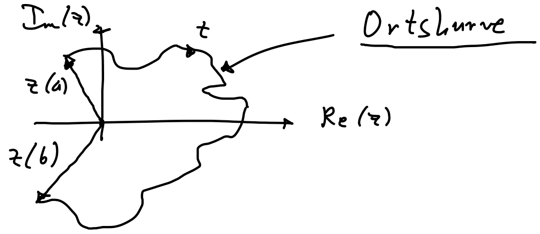
⋮



Def.: Eine Vorschrift: $z : [a, b] \longrightarrow \mathbb{C}$

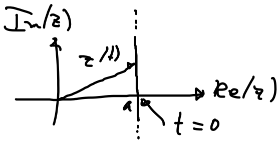
$$t \longmapsto z(t) = x(t) + j\,y(t)$$

heißt <u>komplexwertige Fkt.</u> $z(t)$, der reellen Variablen t.

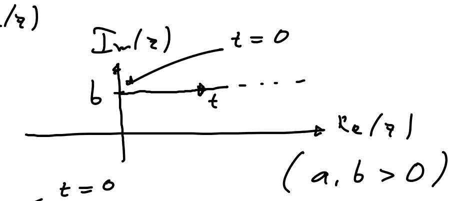


Bem.: $z(t) = x(t) + j\,y(t)$
als Ortskurve in $\mathbb{C}$ ist analog zur Kurve: $\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ in $\mathbb{R}^2$.

Bsp.: (i) $z(t) = a + j\,bt, \quad t \in \mathbb{R}$



(ii) $z(t) = at + jb; \quad t \geq 0$



(iii) $z(t) = 4\,e^{2jt}; \quad t \in [0, \pi]$

$$= 4\left(\underline{\cos(2t)} + j\,\underline{\sin(2t)}\right)$$