# Wiederholung

**LTI-System:** $a_n y^{(n)}(t) + \dots + a_0 y(t) = b_m u^{(m)}(t) + \dots + b_0 u(t)$

$(n > m)$

linear, zeit-invariant

$u(t)$ : Eingang

$y(t)$ : Ausgang

$$u(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$$

$\mathcal{L}(\dots)$: $(a_n s^n + \dots + a_0) Y(s) = (b_m s^m + \dots + b_0) U(s)$

(mit Anfangsbed. $= 0$)

**Transferfk.:**

$$\boxed{G(s) = \frac{Y(s)}{U(s)}} = \frac{b_m s^m + \dots + b_0}{a_n s^n + \dots + a_0}$$

$$U(s) \longrightarrow \boxed{\cdot\, G(s)} \longrightarrow Y(s)$$

**Schrittantwort:**

$u(t) = H(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$ : Schrittantwort

$U(s) = \frac{1}{s}$

$$\longrightarrow \quad G(s) = s\,Y(s) = \mathcal{L}(\underset{\substack{\uparrow \\ \text{gemessen}}}{y'(t)}) + y(0)$$

**Impulsantwort:**

$u(t) = \delta(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$ : Impulsantwort

$U(s) = 1$

$$\longrightarrow \quad G(s) = Y(s) = \mathcal{L}(\underset{\substack{\uparrow \\ \text{(gemessen)}}}{y(t)})$$

**Grenzwerte:**

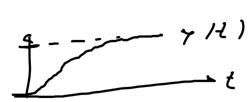
Anfangswert: $\lim\limits_{t \to 0} f(t) = \lim\limits_{s \to \infty} sF(s)$

Endwert: $\lim\limits_{t \to \infty} f(t) = \lim\limits_{s \to 0} sF(s)$

**Stationäre Verstärkung (DC-Gain):** Ist asymptotischer Wert der Schrittantwort.

I.e. Eingang: $u(t) = H(t)$


Ausgang:




Stat. Verstärkung: $\lim\limits_{t \to \infty} y(t)$

Wissen: (aus Schrittantwort) $Y(s) = G(s)U(s) = \dfrac{G(s)}{s}$

$$U(s) = \mathcal{L}(H(t)) = \frac{1}{s}$$

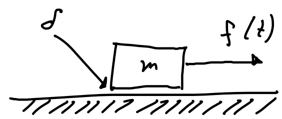
$\longrightarrow$ stat. Verstärkung: $\boxed{\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} G(s)}$

I.e. stat. Verst. direkt aus Transferfkt.

Bsp: Stat. Verst. für System: $G(s) = \dfrac{20(1+3s)}{s^2+7s+10}$ :

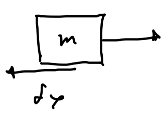
$$\lim_{s\to 0} G(s) = 2$$

Aufgabe: Masse $m$ liegt auf horizontaler Ebene.
Reibung prop. Geschw. (prop. Konstante: $\delta$)
Externe Kraft: $f(t)$.




Ges:
(i) DGL für Geschw. $y(t)$
(ii) Transferfkt. $G(s)$.
(iii) Schrittantwort
(iv) Impulsantwort
(v) Stat. Verstärkung

Lösg: (i)




$F = ma$ wird zu: $my' = -\delta y + f$

$\longrightarrow$ $\underline{my' + \delta y = f}$

(ii) $\mathcal{L}(\ldots) \longrightarrow (ms+\delta)Y = F \longrightarrow G = \dfrac{Y}{F} = \dfrac{1}{ms+\delta}$
(mit $y(0)=0$)

(iii) $f(t) = H(t) \longrightarrow F(s) = \dfrac{1}{s} \longrightarrow Y = GF = \dfrac{1}{s(ms+\delta)} = \dfrac{1}{m}\cdot\dfrac{1}{s(s+\frac{\delta}{m})}$

$$\overset{\text{PBZ}}{=} \frac{1}{m}\left(\frac{m}{\delta s} - \frac{m}{\delta\left(s+\frac{\delta}{m}\right)}\right)$$

$$\longrightarrow y(t) = \mathcal{L}^{-1}(Y(s)) = \frac{1}{\delta}\left(1 - e^{-\frac{\delta}{m}t}\right)$$

(iv) $f(t) = \delta(t) \longrightarrow F(s) = 1 \longrightarrow Y = GF = G = \dfrac{1}{ms+d}$

$$\longrightarrow y(t) = \mathcal{L}^{-1}(Y(s)) = \frac{1}{m}e^{-\frac{\delta}{m}t}$$

(v) $f(t) = H(t) \longrightarrow F(s) = \dfrac{1}{s} \longrightarrow Y = GF = \dfrac{1}{s(ms+\delta)}$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0}\frac{1}{ms+\delta} = \frac{1}{\delta}$$

Aufgabe: Schrittantwort eines Systems sei: $x(t) = 1 - \frac{1}{2}e^{-2t}$

Ges: Transferfkt.

Lösg: $G(s) = \mathcal{L}(x'(t)) + x(0)$

$$= \mathcal{L}(e^{-2t}) + \frac{1}{2} = \frac{1}{s+2} + \frac{1}{2} = \frac{s+4}{2(s+2)}$$

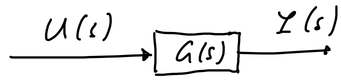
Erinnerung:

$$\begin{array}{ccc} x(t) & \xrightarrow{\mathcal{L}} & X(s) \\ \downarrow & & \downarrow \\ x'(t) & \xrightarrow{\mathcal{L}} & sX(s) - x(0) \end{array}$$

$$G = \frac{X}{U} = sX \qquad u(t) = 1(t) \;;\; U(s) = \frac{1}{s}$$

## Kombination von Transferfkt.

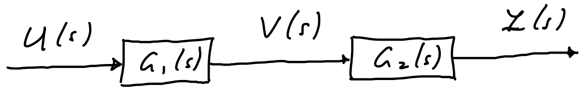
**Einfaches System:**




$$Y(s) = G(s)U(s)$$

**Verknüpfungen:**

**Serie:**

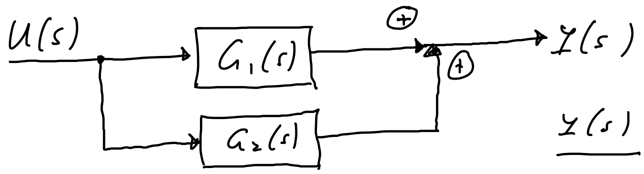



$$V(s) = G_1(s)\,U(s)$$

$$\underline{Y(s)} = G_2(s)\,V(s)$$

$$= \underbrace{G_2(s)G_1(s)}_{G(s):\ \text{Transferfkt. Gesamtsystem}}U(s)$$

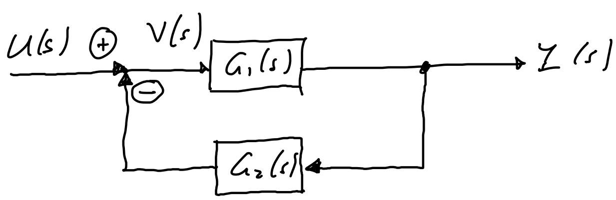
**Parallel:**




$$\underline{Y(s)} = G_1(s)U(s) + G_2(s)U(s)$$

$$= \underbrace{\left(G_1(s) + G_2(s)\right)}_{G(s)} U(s)$$

**Mit Rückführung:**




$$V(s) = U(s) - G_2(s)\,Y(s)$$

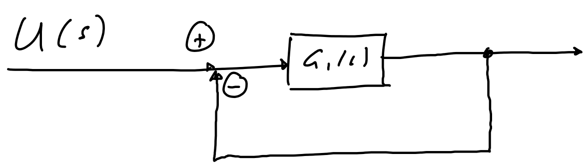
$$Y(s) = G_1(s)\,V(s)$$

$$= G_1(s)\left(U(s) - G_2(s)\,Y(s)\right)$$

$$= G_1(s)U(s) - G_1(s)G_2(s)Y(s) \longrightarrow Y(s) = \frac{G_1(s)U(s)}{1 + G_1(s)G_2(s)}$$

$$\text{I.e.} \quad G(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)}$$

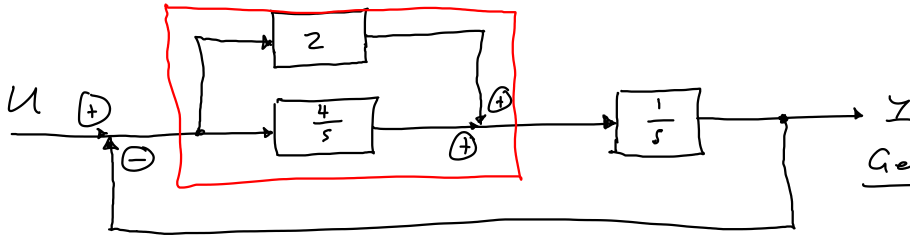
**Spezialfall:**




$$\text{I.e.} \quad G_2(s) = 1$$

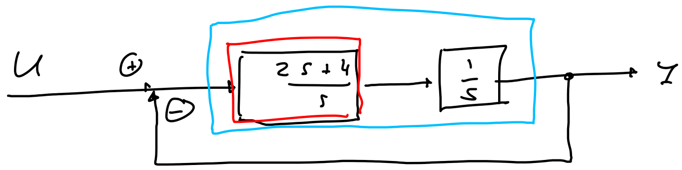
$$\longrightarrow G(s) = \frac{G_1(s)}{1 + G_1(s)}$$

Aufgabe:

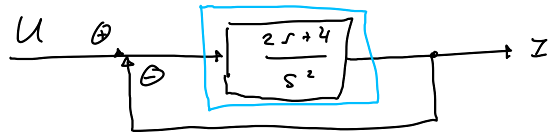



Ges: Transferfkt

Lösg: Äq. zu:




Äq. zu:




$$\longrightarrow \quad \underline{G(s)} = \frac{\frac{2s+4}{s^2}}{1 + \frac{2s+4}{s^2}} = \underline{\frac{2s+4}{s^2+2s+4}}$$