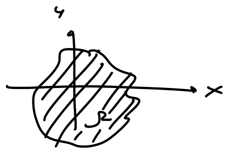
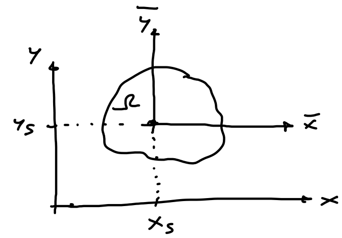
Zu Flächenmomente 2. Ordnung:

Bsp.: $I_x = \iint_\Omega y^2 \, dA \longrightarrow$ Geometrische Widerstand gegen Biegung wenn Moment entlang x-Achse.



Satz von Steiner:



(x_s, y_s) : Schwerpkt.

$$\underline{I_{x_s}} = \iint_\Omega \bar{y}^2 \, dA = \iint_\Omega (y - y_s)^2 \, dA = \iint_\Omega (y^2 - 2yy_s + y_s^2) \, dA$$

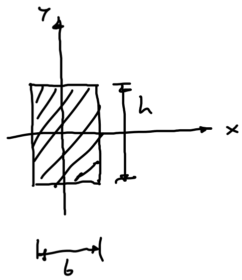
I_x aber bzgl. $\bar{x}, \bar{y}$ - Koord.

$$= \iint_\Omega y^2 \, dA - 2y_s \underbrace{\iint_\Omega y \, dA}_{= y_s|\Omega|} + y_s^2 \underbrace{\iint_\Omega dA}_{= |\Omega|}$$

$$= I_x - 2y_s^2 |\Omega| + y_s^2 |\Omega|$$

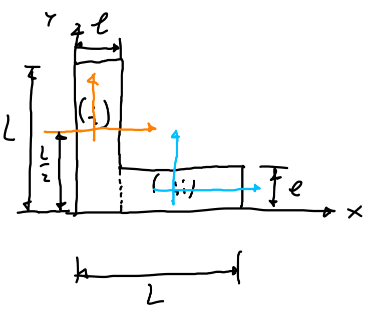
$$= \underline{I_x - y_s^2 |\Omega|}$$

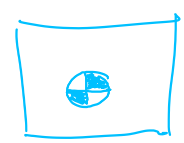
Für



$\longrightarrow I_x = \frac{b h^3}{12}$

Bsp:





$$I_x = I_{x(i)} + I_{x(ii)}$$

$$I_{x(i)} = \frac{e L^3}{12} + \underbrace{L e \left(\frac{L}{2}\right)^2}_{\text{Steiner}} = \frac{L^3 e}{3}$$

$$I_{x(ii)} = \frac{(L-e) e^3}{12} + (L-e) e \left(\frac{e}{2}\right)^2$$

$$= \frac{e^3 (L-e)}{3}$$

$$\longrightarrow \underline{I_x = \frac{1}{3}\left(L^3 e + e^3 (L-e)\right)}$$

Bzgl. SP:

$$y_s = \frac{1}{|\Omega|} \iint_\Omega y \, dA = \frac{1}{|\Omega|} \left(\int_0^L \int_0^e y \, dx \, dy + \int_0^e \int_e^L y \, dx \, dy \right)$$

$$\left[\int_0^e y \int_e^L dx \, dy = \int_0^e y \, \underbrace{x \Big|_e^L}_{L-e} dy = (L-e) \underbrace{\int_0^e y \, dy}_{\frac{y^2}{2}\Big|_0^e = \frac{e^2}{2}} \right]$$

$$= \frac{1}{|\Omega|} \left(e \int_0^L y \, dy + (L-e) \int_0^e y \, dy \right)$$

$$= \frac{1}{|\Omega|} \left(\frac{e L^2}{2} + \frac{(L-e) e^2}{2} \right)$$

$$= \frac{1}{2|\Omega|} \left(e L^2 + (L-e) e^2 \right)$$

$$I_{xs} = I_x - y_s^2 |\Omega|$$

$$= \frac{1}{3}\left(L^3 e + e^3 (L - e) \right) - \underbrace{\frac{1}{4|\Omega|} \times \Big(\underbrace{\underbrace{eL^2}_{m^3} + (L-e)e^2}_{m^6}\Big)^2 \cancel{|\Omega|}}_{m^4}$$