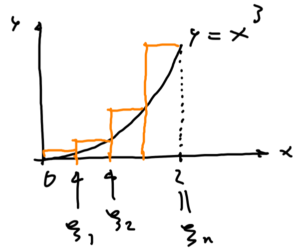
$\boxed{17}$ $\int_0^2 x^3\,dx$, mit Obersummen

Repetition



$\downarrow$

Da wachsende Fkt.

$\longrightarrow$ benötigen Auswertungen der Fkt.

am rechten Rand der Teilintervalle

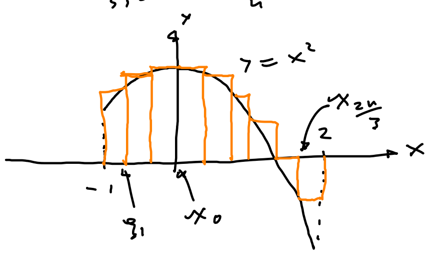
$\longrightarrow \xi_i = \frac{2}{n} i \quad ; \quad i = 1,2,3,\dots,n$

Prüfen: $i=1 \longrightarrow \xi_1 = \frac{2}{n}\cdot 1 = \frac{2}{n}$

$i=n \longrightarrow \xi_n = \frac{2}{n} n = 2$

[Bsp: $\int_{-1}^{2} (-x^2+2)\,dx$ Obersummen

$\xi_i = -1 + \frac{3}{n} i$



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[Bsp: $\int_1^3 x^3\,dx$ mit Obersummen $\longrightarrow \xi_i = 1 + \frac{2}{n} i \quad ; \quad i = 1,2,\dots,n$

Untersummen: $\xi_i = 1 + \frac{2}{n}(i-1)$]

$$S_n = \frac{3}{n}\sum_{i=1}^{n/3} f(\xi_i) + \frac{3}{n}\sum_{k=1}^{\frac{2n}{3}} f(x_k)$$

$\xi_i = -1 + i\frac{3}{n}$; $x_k = (k-1)\frac{3}{n}$

n durch 3 teilbar

$\tilde{n} = 3n$

$\downarrow$

$= \dots$

$\left(\frac{2n}{3} - 1\right)\frac{3}{n} = \frac{2n-3}{3}\cdot\frac{3}{n}$

$= \frac{2n-3}{n}$

$= 2 - \frac{3}{n}$

zurück zur Aufgabe: $\xi_i = \frac{2}{n} i \quad ; \quad i = 1,2,\dots,n$

$$\longrightarrow \overline{S}_n = \frac{b-a}{n}\sum_{i=1}^{n} f(\xi_i)$$

$$= \frac{2}{n}\sum_{i=1}^{n} \underbrace{f\left(\tfrac{2}{n} i\right)}$$

$\longrightarrow f(x) = x^3$

$\longrightarrow f(\xi_i) = \xi_i^3 = \left(\frac{2}{n} i\right)^3$

$$= \frac{2}{n}\sum_{i=1}^{n}\left(\frac{2}{n} i\right)^3$$

$$= \frac{2}{n}\sum_{i=1}^{n}\frac{8}{n^3} i^3 = \frac{16}{n^4}\sum_{i=1}^{n} i^3$$

$$= \frac{16}{n^4}\cdot\frac{1}{4} n^2 (n+1)^2$$

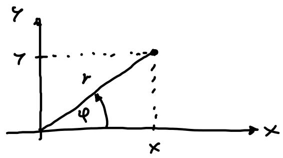
$$= \frac{4}{n^2}(n+1)^2 = \frac{4}{n^2}(n^2+2n+1)$$

$$= 4 + \frac{8}{n} + \frac{4}{n^2}$$

I.e. $\overline{S}_n = 4 + \frac{8}{n} + \frac{4}{n^2} \longrightarrow \underline{\int_0^2 x^3\,dx} = \lim_{n\to\infty} \overline{S}_n = \lim_{n\to\infty}\left(4 + \frac{8}{n} + \frac{4}{n^2}\right) = \underline{4}$

[(ii) $\int_0^2 x^3\,dx = \frac{x^4}{4}\Big|_0^2 = \underline{4}$]

Zu Polarkoord.

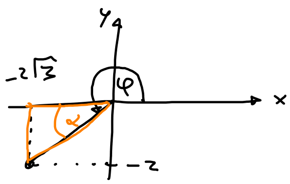


$$\begin{cases} x = r\cos(\varphi) \\ y = r\sin(\varphi)\end{cases}$$

$$\begin{cases} r=\sqrt{x^2+y^2} \\ \tan(\varphi) = \frac{y}{x} \longrightarrow \varphi \end{cases}$$

↑ mit Menschenverstand

Bsp: $(x,y)=(-2\sqrt{3},-2)$

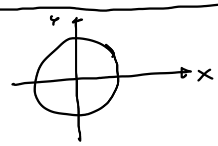


$\tan(\alpha)=\frac{2}{2\sqrt{3}}=\frac{1}{\sqrt{3}}\longrightarrow \alpha=\frac{\pi}{6}$

$\longrightarrow \underline{\varphi} = \alpha + \pi = \underline{\frac{7\pi}{6}}$

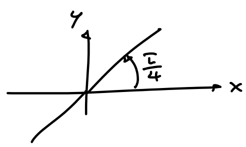
Geom. Objekte in x-y-Ebene:

Kreis: $r=17$



$[x^2+y^2=17^2]$

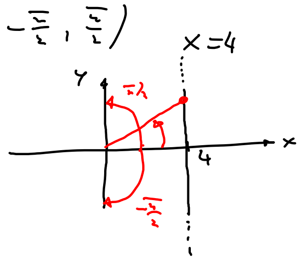
Gerade durch Ursprung: $\varphi=\frac{\pi}{4}$



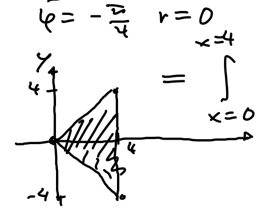
$r=\frac{4}{\cos(\varphi)}$; $\varphi\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

↓

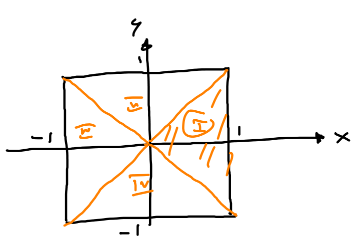
$\underbrace{r\cos(\varphi)}_{=x}=4$



Bsp: $\displaystyle\int_{\varphi=-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_{r=0}^{\frac{4}{\cos(\varphi)}} \dots\, dA = \int_{x=0}^{4}\int_{y=-x}^{y=x}\dots\, dA$



Bsp: $\displaystyle\int_{-1}^{1}\int_{-1}^{1}\dots\, dA = \iint_{I} + \iint_{II} + \iint_{III} + \iint_{IV}$



$\displaystyle= \int_{\varphi=-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_{r=0}^{\frac{1}{\cos(\varphi)}} \dots$

$\displaystyle+ \int_{\varphi=\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_{r=0}^{r=\frac{1}{\sin(\varphi)}}\dots$

$+ \dots$

$+ \dots$

I: $\left[\underbrace{y}_{r\sin(\varphi)} = 1 \longleftrightarrow r=\frac{1}{\sin(\varphi)}\right]$

II

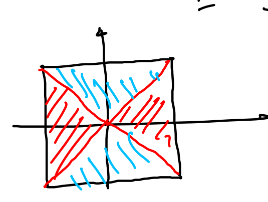
III

IV

[**Bem.:**

Mit $r<0$:

$\displaystyle\int_{\varphi=-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_{r=-\frac{1}{\cos(\varphi)}}^{r=\frac{1}{\cos(\varphi)}} \dots + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_{r=-\frac{1}{\sin(\varphi)}}^{r=\frac{1}{\sin(\varphi)}}$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Bem.: **Flächenelement:** $$\iint_\Omega \ldots \, dA = \iint \ldots \, dx\,dy = \iint \ldots \, r \, dr \, d\varphi$$