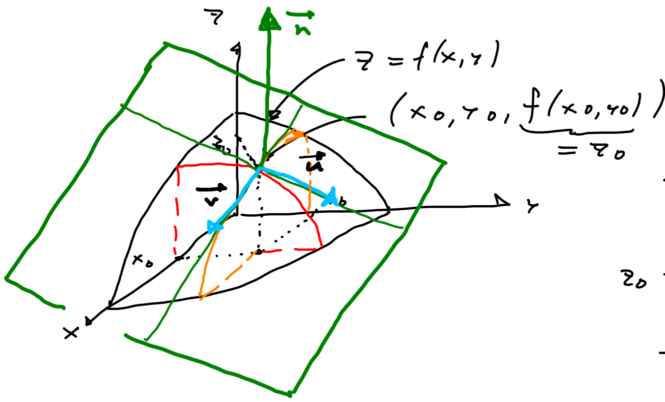
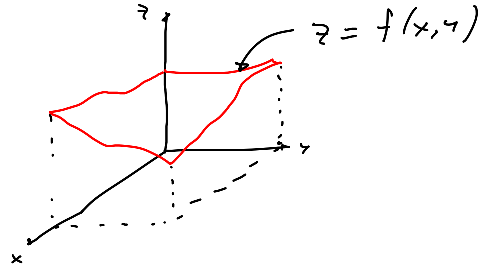
Repetition

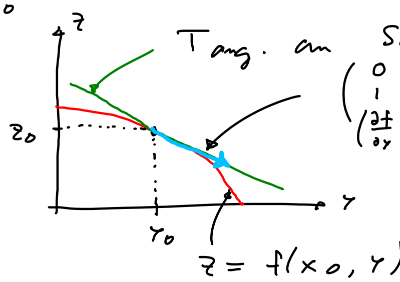
Fkt. von 2 Var.: $f(x,y)$

Graph: $z = f(x,y)$



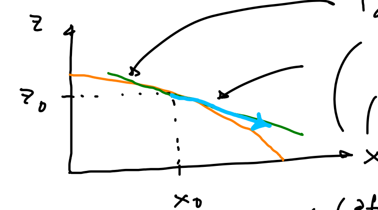


Tang. an Schnittkurve, mit Steigung: $\frac{\partial f}{\partial y}(x_0,y_0)$



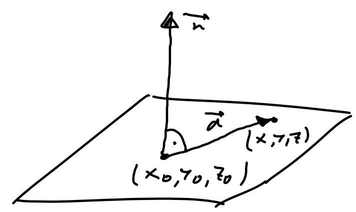
$$\begin{pmatrix} 0 \\ 1 \\ \left(\frac{\partial f}{\partial y}\right)_0 \end{pmatrix} = \vec{u}$$

Schnittkurve $y = y_0$ = konst.: Tang.; Steigung: $\left(\frac{\partial f}{\partial x}\right)_0$



$$\begin{pmatrix} 1 \\ 0 \\ \left(\frac{\partial f}{\partial x}\right)_0 \end{pmatrix} = \vec{v}$$

$$\vec{n} = \vec{u} \times \vec{v} = \begin{pmatrix} 0 \\ 1 \\ \left(\frac{\partial f}{\partial y}\right)_0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ \left(\frac{\partial f}{\partial x}\right)_0 \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial f}{\partial x}\right)_0 \\ \left(\frac{\partial f}{\partial y}\right)_0 \\ -1 \end{pmatrix}$$



Ebenengl.: $\vec{n} \cdot \underbrace{\begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}}_{= \vec{d}} = 0$ ist:

$$\left(\frac{\partial f}{\partial x}\right)_0 (x - x_0) + \left(\frac{\partial f}{\partial y}\right)_0 (y - y_0) - (z - z_0) = 0$$

oder:

$$\underline{z - z_0 = \left(\frac{\partial f}{\partial x}\right)_0 (x - x_0) + \left(\frac{\partial f}{\partial y}\right)_0 (y - y_0)}$$

Herleitung mit Gradient:

Fkt. von 2 Var.: $f(x,y)$

Fkt. von 3 Var: $g(x,y,z) = f(x,y) - z$

$\longrightarrow$ Graph von $f(x,y)$ (i.e. $z = f(x,y)$) ist Niveaufläche von $g(x,y,z)$, nämlich $g(x,y,z) = 0$

$\updownarrow$

($f(x,y) - z = 0$

i.e. $f(x,y) = z$)

Es gilt: Gradient steht $\perp$ auf Niveauflächen

$$\longrightarrow \quad \vec{n} = \vec{\nabla} g = \begin{pmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \\ \frac{\partial g}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ -1 \end{pmatrix}$$