Wiederholung:

Fkt. von mehreren Variablen

$$f: D \longrightarrow \mathbb{R}$$
$$(x,y) \longmapsto z = f(x,y) \qquad \text{Hier: } D \subset \mathbb{R}^2$$

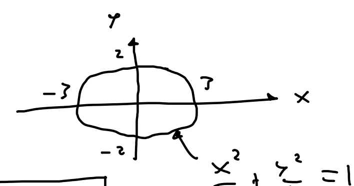
Bsp: $U = f(R, I) = RI$

Betrachten: $f(x,y) = \log(36 - 4x^2 - 9y^2)$

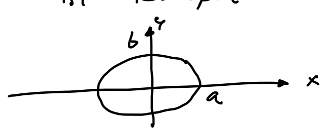
Def.-Bereich: $36 - 4x^2 - 9y^2 > 0$

$$\longrightarrow \quad 4x^2 + 9y^2 < 36$$
$$\longrightarrow \quad \underline{\frac{x^2}{9} + \frac{y^2}{4} < 1}$$

Betrachten: $\frac{x^2}{9} + \frac{y^2}{4} = 1 \longrightarrow$ Ellipse : $x = 0 \longrightarrow y = \pm 2$, $y = 0 \longrightarrow x = \pm 3$



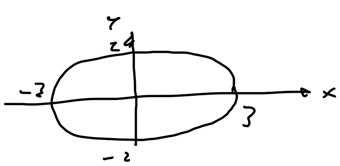
Allgemein: $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ ist Ellipse mit Halbachsen } a \,\&\, b}$

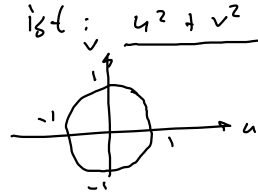


Anderer Zugang zur Ellipse:

$\frac{x^2}{9} + \frac{y^2}{4} = 1$ in neuen Koord.: $u = \frac{x}{3}$, $v = \frac{y}{2}$

ist: $\underline{u^2 + v^2 = 1} \longrightarrow$ Kreis



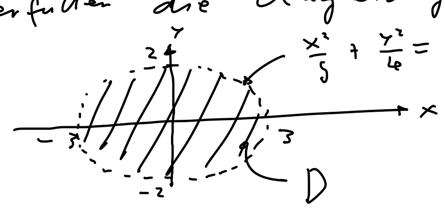


Def.-Bereich: $D = \{(x,y) \in \mathbb{R}^2 \mid \frac{x^2}{9} + \frac{y^2}{4} < 1\}$

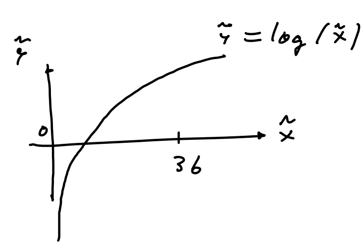
$(x,y) = (0,0)$ erfüllt die Ungleichung

$\longrightarrow$ alle Punkte im Innern der Ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ erfüllen die Ungleichung.

$\longrightarrow$



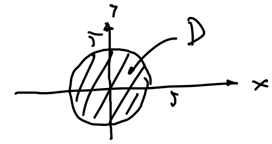
Wertebereich: $f(x,y) = \log(\underbrace{36 - 4x^2 - 9y^2}_{\in (0, 36]})$



$$\longrightarrow \quad W = (-\infty,\ \log(36)\,]$$

Aufgabe: Ges.: Def.- & Wertebereich von $f(x,y) = \sqrt{25 - x^2 - y^2}$

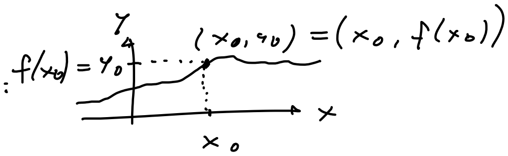
Lösg.: $D = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 25\}$



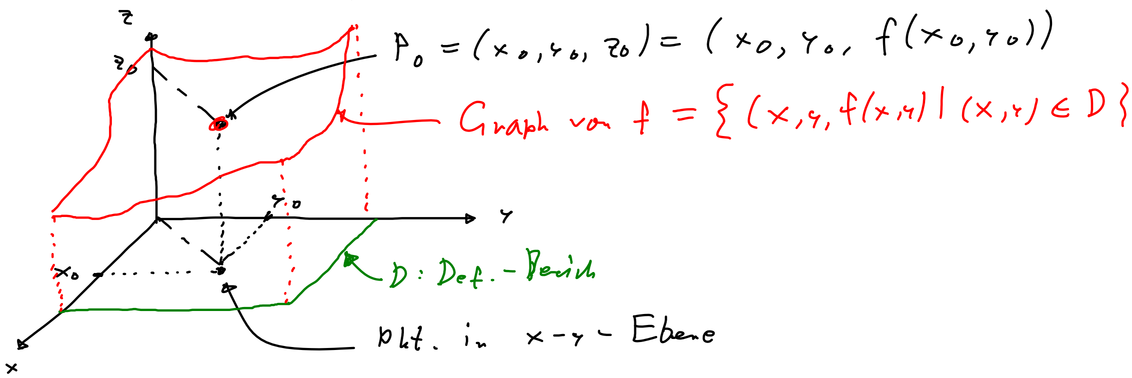
$$W = [0, 5]$$

Grafische Darstellung

Erinnerung:



Sei $z = f(x,y)$ gegeben.



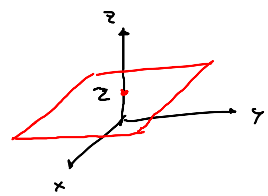
Aufgabe: Man skizziere (den Graphen):

(i) $f(x,y) = 2$

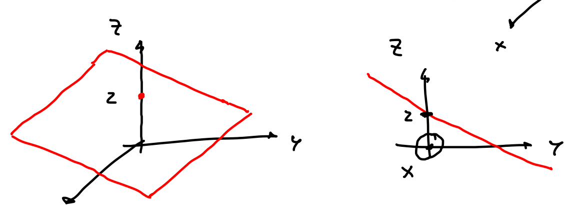
(ii) $f(x,y) = -y + 2$

(iii) $f(x,y) = 3 - \frac{1}{4}(3x + 6y)$

Lösg.: (i) $z = 2$: Horizontale Ebene:



(ii)



(iii) $z = 3 - \frac{1}{4}(3x + 6y) \quad / \cdot 4$

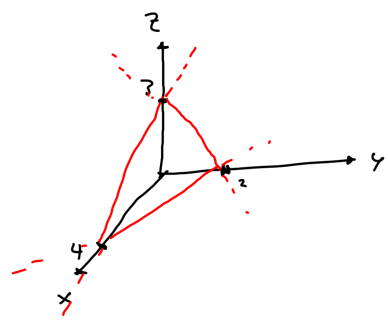
$\longrightarrow \quad 4z = 12 - 3x - 6y$

$\longrightarrow \quad 3x + 6y + 4z = 12$

$x, y = 0 \longrightarrow z = 3$

$x, z = 0 \longrightarrow y = 2$

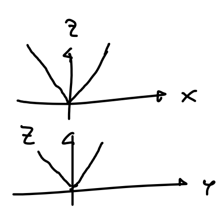
$y, z = 0 \longrightarrow x = 4$

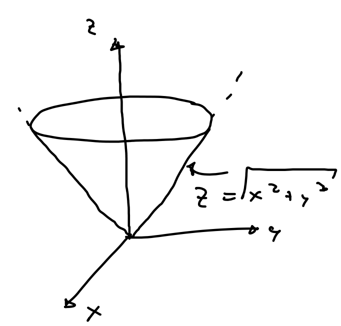


Bsp: $f(x,y) = \sqrt{x^2 + y^2}$

$y = 0 \longrightarrow z = \sqrt{x^2} = |x|$

$x = 0 \longrightarrow z = \sqrt{y^2} = |y|$



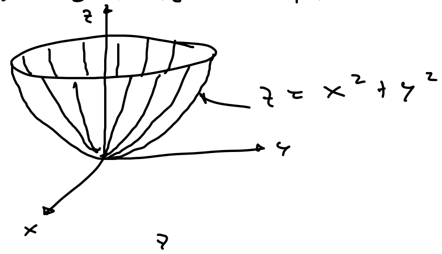


Allgemein sind Fkt. vom Typ: $f(\sqrt{x^2+y^2})$

<u>rotationssymm.</u>! <u>Rezept:</u> Man zeichne $f(\sqrt{x^2+y^2})$ mit $y=0$ für $x>0$.
Dann rotiert man diese Kurve um z-Achse.

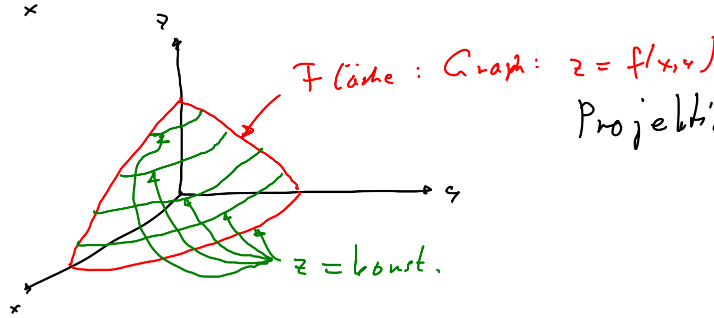
<u>Aufgabe:</u> Man zeichne: $f(x,y) = x^2+y^2$.

<u>Lösg.:</u>

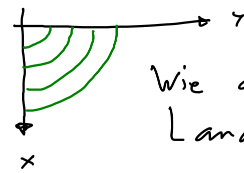


(Rotationsparaboloid)

<u>Höhenlinien:</u>



Projektion auf x-y-Ebene:



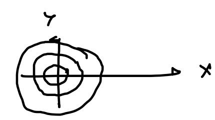
Wie auf Landkarte!

Gleichung Höhenlinie mit $z = C$:

$$\boxed{f(x,y) = C}$$

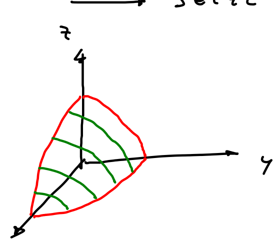
Bsp: Höhenlinie zu $z = f(x,y) = x^2+y^2$

$\longrightarrow x^2+y^2 = C \longrightarrow$ Kreis mit Radius $\sqrt{C}$ mit Zentrum im Ursprung

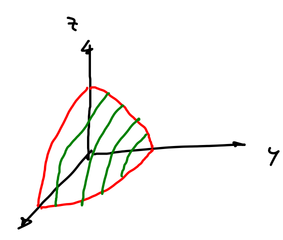


<u>Schnittkurven:</u>

(i) Parallel zur y-z-Ebene:
$\longrightarrow$ setze <u>$x = C$</u>



(ii) Parallel zur x-z-Ebene
$\longrightarrow$ setze <u>$y = C$</u>



<u>Aufgabe:</u> Man finde zu $f(x,y) = \frac{y}{x}$

(i) Höhenlinien zu $z = 0, 1, 2$ (im Bereich: $x>0$; $y \geq 0$)

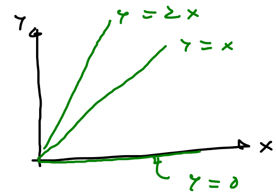
(ii) Schnittkurven parallel zur x-z-Ebene zu $y = 0, 1, 2, 3$

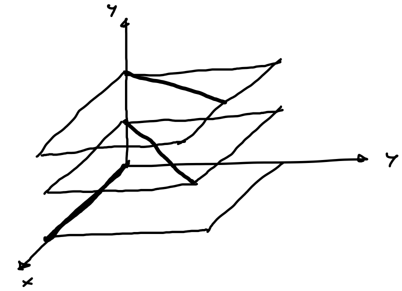
Lösg.:

(i) $z = 0 \longrightarrow \frac{y}{x} = 0 \longrightarrow \underline{y = 0}$

$z = 1 \longrightarrow \frac{y}{x} = 1 \longrightarrow \underline{y = x}$

$z = 2 \longrightarrow \frac{y}{x} = 2 \longrightarrow \underline{y = 2x}$





(ii) $y = 0 \longrightarrow z = \frac{0}{x} = 0$

$y = 1 \longrightarrow z = \frac{1}{x}$

$y = 2 \longrightarrow z = \frac{2}{x}$

$y = 3 \longrightarrow z = \frac{3}{x}$