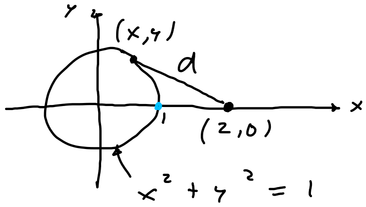
zu 175



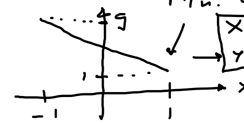
$D(x) = \text{Abstand}^2 = d^2 = (2-x)^2 + y^2$

$\quad = (2-x)^2 + 1 - x^2$

(Nebenbed.: $x^2+y^2=1$
$\longrightarrow y^2 = 1-x^2$)

$\quad = 4 - 4x + x^2 + 1 - x^2$

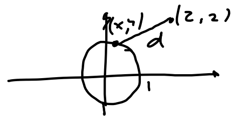
$\quad = \underline{5 - 4x}$



Min. bei $\boxed{x=1,\ y=0}$

(Bem. Implizite Ableitung: $F(x,y) = 3x^2 - 17\sin(y)$
Kurve in $\mathbb{R}^2$: $F(x,y)=0$
Ges: $\frac{dy}{dx}$)

Anderer Pkt.: $(2,2)$



$D = d^2 = (x-2)^2 + (y-2)^2$

$\quad = x^2 - 4x + 4 + y^2 - 4y + 4$

NB: $x^2+y^2=1 \longrightarrow y^2 = 1-x^2$

$\longrightarrow y = \pm\sqrt{1-x^2}$

oberer Halbkreis: $D(x) = \cancel{x^2} - 4x + 4 + 1 - \cancel{x^2} - 4\sqrt{1-x^2} + 4$

$\quad = 9 - 4x - 4\sqrt{1-x^2}$

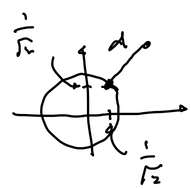
$\longrightarrow \frac{dD}{dx}(x) = -4 - \frac{2}{\sqrt{1-x^2}}(-2x) = -4 + \frac{4x}{\sqrt{1-x^2}} \overset{!}{=} 0$

$\longrightarrow \boxed{\sqrt{1-x^2} = x}$ $\quad / (\ldots)^2$

$1 - x^2 = x^2$

$1 = 2x^2$

$\longrightarrow x = \pm\frac{1}{\sqrt{2}}$



$\longrightarrow x = \frac{1}{\sqrt{2}} \longrightarrow y = \frac{1}{\sqrt{2}}$ Min.

(unterer Halbkreis: analog $\longrightarrow \ldots \longrightarrow x = -\frac{1}{\sqrt{2}},\ y = -\frac{1}{\sqrt{2}}$ Max.)

zu 131

$f(x) = xe^{-x} \longrightarrow \frac{df}{dx}(x) = e^{-x} + xe^{-x}(-1)$

$\quad = \underbrace{e^{-x}}_{\neq 0}(1-x) \overset{!}{=} 0 \quad / : e^{-x}$

$1 - x = 0 \longrightarrow \underline{x = 1}$

$\frac{d^2f}{dx^2}(x) = -e^{-x}(1-x) + e^{-x}(-1)$

$\quad = e^{-x}(-1+x-1) = e^{-x}(x-2)$

$\longrightarrow \frac{d^2f}{dx^2}(1) = e^{-1}(1-2) = -e^{-1} < 0$

$\longrightarrow$ Max. bei $\underline{x=1}$, $\underline{y = \frac{1}{e}}$

Ableitung $\arcsin(x)$: $1 = \frac{d}{dx}x = \frac{d}{dx}f(f^{-1}(x)) = \frac{df}{dx}(f^{-1}(x))\frac{df^{-1}}{dx}(x)$

$\longrightarrow \frac{df^{-1}}{dx}(x) = \frac{1}{\frac{df}{dx}(f^{-1}(x))}$

$\longrightarrow \quad f(x) = \sin(x) \longrightarrow f^{-1}(x) = \arcsin(x)$

$\quad \hookrightarrow \frac{df}{dx}(x) = \cos(x)$

$\cos(x) = \sqrt{1 - \sin^2(x)}$

$\downarrow$ ("+" – Lösg., da $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$)

I.e.

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\cos(\arcsin(x))} = \frac{1}{\sqrt{1 - \sin^2(\arcsin(x))}}$$

$$= \frac{1}{\sqrt{1 - (\sin(\arcsin(x)))^2}}$$

[Notation: $\sin^2(x) = (\sin(x))^2$]

$$= \frac{1}{\sqrt{1 - x^2}}$$

$$a^x = e^{x\log(a)}$$

zu 137 $\quad h(x) = x^x \overset{\downarrow}{=} e^{x\log(x)}$

$$\longrightarrow \frac{dh}{dx}(x) = \frac{d}{dx}\left(e^{x\log(x)}\right) = e^{x\log(x)}\frac{d}{dx}\left(x\log(x)\right)$$

$$= \underbrace{e^{x\log(x)}}_{=x^x}\Big(\log(x) + \overbrace{x\tfrac{1}{x}}^{=1}\Big)$$

$$= x^x\left(\log(x) + 1\right)$$

zu 138 (ix) $\quad g(u) = (u\log(u))^{10}$

$$\frac{dg}{du}(u) = 10\left(u\log(u)\right)^9 \cdot \frac{d}{du}\left(u\log(u)\right)$$

$$= 10\left(u\log(u)\right)^9\left(\log(u) + 1\right)$$

zu 148 $\quad$ Ges: $\frac{1}{n!}\frac{d^n p_n}{dx^n}$, wobei $p_n(x) = \sum_{k=0}^{n} a_k x^k$

$$= a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1}x^{n-1} + a_n x^n$$

$$\frac{d^n p_n}{dx^n} = \frac{d^n}{dx^n}\sum_{k=0}^{n} a_k x^k = \sum_{k=0}^{n}\frac{d^n}{dx^n}(a_k x^k)$$

$$\underset{\uparrow}{=} \frac{d^n}{dx^n}(a_n x^n) = a_n\frac{d^n}{dx^n}x^n = a_n\frac{d^{n-1}}{dx^{n-1}}(n x^{n-1})$$

$\frac{d^n}{dx^n}x^k = 0$ für $k < n$

$$= a_n\frac{d^{n-2}}{dx^{n-2}}\left(n(n-1)x^{n-2}\right)$$

$$\vdots$$

$$= a_n\, n(n-1)(n-2)(n-3)\cdots 2\cdot 1\,\underbrace{x^0}_{=1}$$

$$= a_n\, n! \longrightarrow \frac{1}{n!}\frac{d^n p_n}{dx^n} = a_n$$

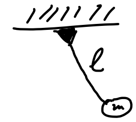
Bsp: $\quad p_3 = 17 + 11x - 2x^2 + 5x^3$

$$\frac{d^3 p_3}{dx^3}(x) = 5\frac{d^3}{dx^3}(x^3)$$

$$= 5\frac{d^2}{dx^2}(3x^2) = 5\frac{d}{dx}(3\cdot 2x) = 5\cdot 3\cdot 2\cdot 1 = 5\cdot 3!$$

$$\longrightarrow \frac{1}{3!}\frac{d^3 p_3}{dx^3} = 5$$

zu 185 $T(\ell) = 2\pi\sqrt{\frac{\ell}{g}}$



$$T(\ell) \approx T(\ell_0) + \frac{dT}{d\ell}(\ell_0)\underbrace{(\ell-\ell_0)}_{\Delta\ell} \qquad [\text{Linearisierung}]$$

$$\Delta T = T(\ell) - T(\ell_0)$$

$\longrightarrow$ Ges:
$$\frac{\Delta T}{T_0} \approx \frac{1}{T}\frac{dT}{d\ell}(\ell_0)\,\Delta\ell = \frac{1}{T_0}\,\pi\frac{1}{\sqrt{\ell_0 g}}\,\Delta\ell$$

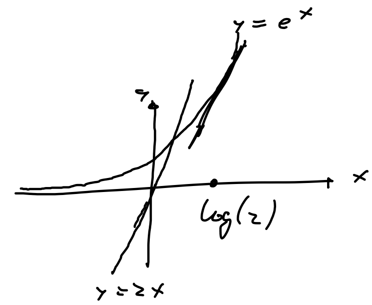
$$\left(\frac{d}{d\ell}\left(2\pi\sqrt{\frac{\ell}{g}}\right) = \pi\frac{1}{\sqrt{\ell g}}\right)$$

$$= \frac{1}{2\pi\sqrt{\frac{\ell_0}{g}}}\,\pi\frac{1}{\sqrt{\ell_0 g}}\,\Delta\ell = \frac{1}{2\ell_0}\Delta\ell = \frac{1}{2}\cdot\frac{\Delta\ell}{\ell_0}$$

zu 124 $f(x) = e^x$ Tang. parallel zu $y = 2x$

$$\frac{df}{dx}(x) = e^x \qquad \frac{dy}{dx} = 2$$

$$\longrightarrow 2 = e^x \longrightarrow x = \log(2)$$

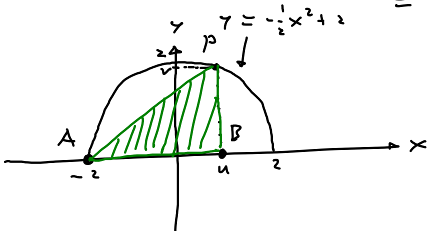


zu 195 (ii)
$$\frac{d}{dx}\left(f(x)\boxed{(g(x))^3}\right) = \frac{df}{dx}(x)(g(x))^3 + f(x)\,3(g(x))^2\frac{dg}{dx}(x)$$

bei $x = 0$
$$\underset{x=0}{=} f'(0)(g(0))^3 + f(0)\,3(g(0))^2 g'(0)$$

$$= 5\cdot 1^3 + 1\cdot 3\cdot 1^2\,\frac{1}{3} = 5 + 1 = 6$$

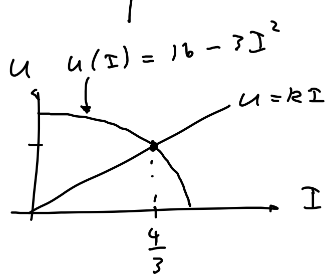
zu 181



$$A(u) = \frac{(u+2)v}{2} = \frac{(u+2)\left(-\frac{1}{2}u^2+2\right)}{2}, \qquad v = -\frac{1}{2}u^2+2$$

$$\frac{dA}{du}(u) = \ldots \overset{!}{=} 0 \longrightarrow u_\pm = \ldots$$

zu 182



$U = RI$

$$P = \underset{U(I)}{U}I = (16 - 3I^2)I = 16I - 3I^3$$

$$\longrightarrow \frac{dP}{dI} = 16 - 9I^2 \overset{!}{=} 0$$

$$16 = 9I^2$$

$$\longrightarrow \underline{I = \frac{4}{3}}$$

$$\longrightarrow U = 16 - 3\left(\frac{4}{3}\right)^2 = 16 - \frac{16}{3} = \frac{32}{3}$$

$$\longrightarrow \underline{R} = \frac{U}{I} = \frac{\frac{32}{3}}{\frac{4}{3}} = \underline{8}$$

Zur <u>Notation</u>: $f' = \frac{df}{dx}$; $f''' = \frac{d^3f}{dx^3}$; $f^{(n)} = \frac{d^nf}{dx^n}$

Zu 154 (iii) $e^x \sin(y) - e^{-y} \cos(x) = 0$

$$\frac{d}{dx} \longrightarrow \quad e^x \sin(y) + e^x \cos(y) \frac{dy}{dx} - e^{-y}\left(-\frac{dy}{dx}\right) \cos(x) - e^{-y}(-\sin(x)) = 0$$

$$\frac{dy}{dx}\left(e^x \cos(y) + e^{-y} \cos(x)\right) = -e^x \sin(y) - e^{-y} \sin(x)$$

$$\longrightarrow \frac{dy}{dx} = \frac{-e^x \sin(y) - e^{-y} \sin(x)}{e^x \cos(y) + e^{-y} \cos(x)}$$

Mit $y = f(x)$:

$$e^x \sin(f(x)) - e^{-f(x)} \cos(x) = 0$$

$\frac{d}{dx} \ldots$

$$\frac{d}{dx} \sin(y(x)) = \cos(y(x)) \cdot \frac{dy}{dx}(x)$$

Kettenregel