Wiederholung

Def.: $$\boxed{\frac{df}{dx}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Geometrisch: Steigung Tang. an $y = f(x)$.

Physikalisch: Geschwindigkeit

(Position: $f(t)$

Geschw.: $\frac{df}{dt}(t)$)

Bsp: $f(x) = x^2$:

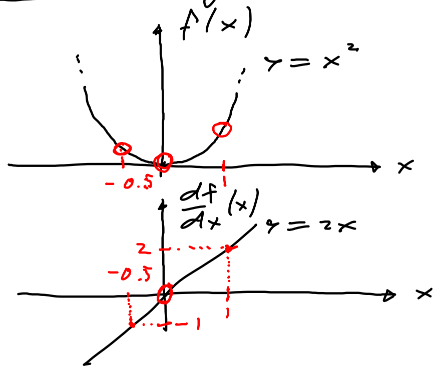
$$\boxed{\frac{d}{dx}x^2} = \frac{df}{dx}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0}(2x + h) = \boxed{2x}$$

Anschauung:



Ableitung elementarer Funktionen

(i) $f(x) = C$: Konstant

$$\frac{df}{dx}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{C - C}{h}$$

$$= \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = 0$$

I.e. $\boxed{\frac{d}{dx} C = 0}$ $\left(\text{Bsp: } \frac{d}{dx} 17 = 0\right)$

(ii) $f(x) = ax$ (a: Konst.)

$$\frac{df}{dx}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{a(x+h) - ax}{h} = \lim_{h \to 0} \frac{\cancel{ax} + ah - \cancel{ax}}{h}$$

$$= \lim_{h \to 0} \frac{ah}{h} = \lim_{h \to 0} a = a$$

I.e. $\boxed{\frac{d}{dx}(ax) = a}$ (Bsp: $\frac{d}{dx}(3x) = 3$)

(iii) $f(x) = x^2$ Haben gesehen: $\boxed{\frac{d}{dx} x^2 = 2x}$

(iv) $f(x) = x^3$:

1

1 1

1 2 1

1 3 3 1

$$\frac{df}{dx}(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2 h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h\to 0} \left(3x^2 + 3xh + h^2\right)$$

$$= 3x^2 \qquad \boxed{\frac{d}{dx} x^3 = 3x^2}$$

(v) $f(x) = x^n$ ["Geraten": $\frac{d}{dx} x^n = n x^{n-1}$]

$$\frac{df}{dx}(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\boxed{(x+h)^n} - \boxed{x^n}}{h}$$

$$(x+h)^n = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \binom{n}{3} x^{n-3} h^3 + \dots + \binom{n}{n-1} x h^{n-1} + \binom{n}{n} h^n$$

$$= x^n + n x^{n-1} h + (\dots) h^2 + \dots + h^n$$

$$\longrightarrow \frac{df}{dx}(x) = \lim_{h\to 0} \frac{\boxed{\cancel{x^n} + n x^{n-1} h + \dots h^2 + \dots + h^n} - \boxed{\cancel{x^n}}}{h}$$

$$= \lim_{h\to 0} \left(n x^{n-1} + \dots h + \dots h^2 + \dots + h^{n-1}\right)$$

$$= n x^{n-1}$$

I.e. $\boxed{\frac{d}{dx} x^n = n x^{n-1}}$ $\boxed{n = 1, 2, 3, \dots}$

($\underline{n = 0:}$ $x^0 = 1$

$\frac{d}{dx} 1 = 0 \cdot x^{-1}$)

<u>Aufgabe:</u>